Determining How Many Words/Numbers Out Of A Given Set:

A. Many questions will give you a set of numbers or letters and ask how many numbers or how many words can be made from a set.

Ex [1] How many 6 letter words, real or imaginary, can be made from [N, U, M, B, E, R]?

Ex [2] How many 4 digit numbers can be made out of [1, 2, 3, 4]?

B. Dealing with letters:

1. If all the letters are distinct then the answer is n! where n is the number of letters.

   Ex [1] How many 6 letter words, real of imaginary, can be made from [N, U, M, B, E, R]?

   a. Since all the letters are distinct, we know the answer is 6! = 720.
   b. The reason is because for the first letter we have 6 choices. The second letter we only have 5 choices. The third letter we only have 4 choices and so on.
   c. So the answer is 6 x 5 x 4 x 3 x 2 x 1.

2. If all the letters are not distinct, then the answer changes. The answer becomes n!/a!, where a is the number of each indistinct letter.

   Ex [2] How many 5 letter words, real or imaginary, can be made from [S, C, O, T, T]?

   a. The answer is $^{5!}/_{2!} = {}^{120}/_{2} = 60$.
   b. The reason is because we have 2 T's. This limits are possibilities because we can have the word - STTOC and STTOC and they are the same word.

   If we were given the letters [S, S, O, T, T] then the answer would become $^{5!}/_{2!*2!}$, because we have 2 S's and 2 T's. If we were given [S, S, S, O, T] the answer would be $^{5!}/_{3!}$, because we have 3 S's.

C. Dealing with numbers:

1. If all the numbers are distinct, AND one of them is not a 0, then the answer is n! where n is the number of digits given.

   Ex [1] How many 4 digit numbers can be made using [1, 2, 3, 4]?

a. The answer is 4! = 24.
b. The reason is because we have 4 choices for the first digit, 3 for the second, and so on.
c. This means there are 4 x 3 x 2 x 1 possibilities.

2. If all the numbers are not distinct, AND one of them is not a 0, then the answer is $^{n!}/_{a!}$, where a is the number of each distinct digit.

Ex [2] How many 5 digit numbers can be made using [9, 3, 5, 7, 9]?

a. Because we have 2 9' s the answer is $^{5!}/_{2!}$ or 60.
b. The reason why is because the number 99753 and 99753 are the same number.

Like above, when dealing with letters, if we had the digits [9, 3, 3, 7, 9] the answer would be $^{5!}/_{2!*2!}$ and if we had [9,9,3,7,9] the answer would be $^{5!}/_{3!}$.

3. If all the numbers are distinct but one of the numbers is a 0, then the answer becomes: (n-1)*(n-1)!, where n is the number of digits. Instead of memorizing a formula, think through it logically. See below.

Ex [3] How many 4 digit numbers can be made using [3, 2, 1, 0]?

a. First, ask yourself how many possibilities are there for the 1st digit. There are only 3, not 4, since 0 cannot start a number.
b. The number of possibilities for the 2nd digit is also 3, since one number was taken out to be placed in the 1st digit. Then there are 2 possibilities for the 3rd and only 1 possibility for the last.
c. This means the probability is: 3 x 3 x 2 x 1 = 18.

4. If the numbers are not all distinct and you have at least 1 zero, then the answer is changed a little. You should follow step (3) above and work the problem as if they were all distinct. Then, in the end, divide by n! where ' n' is the number of repeated digits. See below.

Ex [4] How many 6 digit numbers can be made using [1, 2, 2, 2, 4, 0]?

a. If we treated all of these as distinct numbers then we would have an answer of 5 x 5 x 4 x 3 x 2 x 1 or 600. See step 3.
b. Since we have the digit 2 repeated 3 times, we need to divide by 3! or 6.
c. So the answer is $^{600}/_{6}$ = 100.