Working With Combinations:

A. Combinations are often confused with permutations.

1. A combination can be thought of as a small group of a collection of objects.

2. In other words, if we had 4 people: A,B,C,D, one possible combination of 3 people could be ABC or ABD.

3. Combinations come in many different forms.

   a. Some forms are obvious:

$${}_nC_r \text{ or } C(n,r)$$

   b. Other forms are not so obvious and are written as word problems making them somewhat difficult to recognize:

Ex [1] How many 2-member committees can be formed from 6 people?

Ex [2] How many lines are determined by 4 points, no 3 of which are collinear?

Ex [3] How many triangles can be formed using 3 vertices of a regular hexagon?

1) One way to know if we are dealing with permutations or combinations, is to answer one question: Does the order matter?

2) If the answer is yes, then we will be using combinations, not permutations.

Ex [1] How many 2-member committees can be formed from 6 people?

a. We have 6 people: A, B, C, D, E, and F.

b. Can we have a committee of AB and BA and count this as 2 committees or is this just one? Since we are talking about the same people this just counts as 1, so the order does matter.

Ex [2] How many lines are determined by 4 points, no 3 of which are collinear?

a. We have 4 points: A, B, C, and D.

b. If we draw a line through segment AB and through segment BA, is this two different lines, or just 1? It is only one, so order does matter.

Ex [3] How many triangles can be formed using 3 vertices of a regular hexagon?

a. We have 6 points: A, B, C, D, E, and F.

b. If we create a triangle ABC, ACB, BAC, BCA, CAB, and CBA, are these considered different triangles or the same triangles? They are the same, so order does matter.

B. How to calculate combinations:

1. This method uses factorials.

2. There are ALWAYS fewer combinations than permutations.

3. $C(n,r) = \dfrac{n!}{(n-r)!*r!}$

Ex [1] ${}_3C_2$ = __________.

a. ${}^{3!}/_{[(3-2)!*2!]} = {}^{3!}/_{1!*2!}$
b. ${}^{3!}/_{1!*2!} = {}^{3*2*1}/_{1*2*1} = 3.$
c. The answer is 3.

Ex [2] How many triangles can be formed using 3 vertices of a regular octagon?

a. First, notice we need to calculate C(8,3) since an octagon has 8 vertices.

b. ${}^{8!}/_{(8-3)!*3!} = {}^{8*7*6*5*4*3*2*1}/_{[5*4*3*2*1]*[3*2*1]}$

c. ${}^{8*7*6*5*4*3*2*1}/_{[5*4*3*2*1]*[3*2*1]} = {}^{8*7*6}/_{6}$

d. ${}^{8*7*6}/_{6} = 8 * 7 = 56.$

e. The answer is 56.

C. Notice as in Ex [2] part c, there will always be numbers that cancel each other out, making the problem much easier. It is too hard to calculate 8!, then divide by 5!*3!. You should learn to recognize how to do this to make the problem faster and easier.

D. One property of combinations is that C(n,r) = C(n,n-r). So sometimes we might encounter a problem that looks like:

Ex [3] ${}_7C_3 = {}_7C_n$, n … 3, then n = __________.

a. For problems of this nature simple subtract 3 from 7.

b. The answer is 7-3 = 4.