## **Probability With Coins:**

- A. Coins are a good beginning for learning probability since you are faced with only 2 choices: heads or tails.
- B. Below is a diagram of a coin after it is tossed 3 times. This will be used as an example to explain how to use probability in relation to coins:



- C. There are several things that we can gather from looking at this diagram.
  - 1. The number of possible combinations after 3 flips is  $2^3$  or 8. In general, the number of possible combinations after n flips is  $2^n$ .
  - 2. The probability of getting heads or tails on each successive flip is 1/2.
  - 3. The probability of getting all heads (or all tails) after 3 flips is  $1/_{23}$  which is  $1/_8$ . In general, the probability of getting all heads (or all tails) after n flips is  $1/_{2n}$ .
  - 4. A more complicated concept is how to determine the probability of getting 2 heads and 1 tails (or 2 tails and 1 heads). From counting we know the answer is  ${}^{3}/_{8}$ , but what if we had more than 3 flips? In general, we can calculate the probability of getting x heads and y tails after r flips by using the following formula:

$$P = \frac{C(r,x)}{2^r} \text{ or } \frac{C(r,y)}{2^r}$$

- D. Let's look at some examples. You will need to be familiar with combinations.
  - Ex [1] A coin is flipped 5 times. Find the probability of getting 3 heads and 2 tails.
    - a. First, we know the denominator is  $2^5 = 32$ .
    - b. To find the numerator, we need to calculate C(5,3) [or C(5,2)].
    - c. C(5,3) = 10.
    - d. The answer is  ${}^{10}/_{32} = {}^{5}/_{16}$ .

Ex [2] A coin is tossed 4 times. Find the probability of getting at least 2 heads.

- a. This example is a little harder as we can have 2 heads and 2 tails, 3 heads and 1 tail, or 4 heads and no tails.
- b. This means we will have to calculate each probability and add it together.
- c. The probability of each one successively is:  $C(4,2) + C(4,3) + C(4,4)/_{24}$ . d. This is  $^{6+4+1}/_{16} = ^{11}/_{16}$ . e. The answer is  $^{11}/_{16}$ .