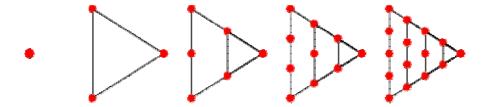
## **Triangular Numbers:**



- A. Triangular numbers are numbers that create triangles. In other words 1, 3, 6, 10, 15, etc
- B. Triangular numbers can be calculated by 1, 1+2, 1+2+3, 1+2+3+4, etc.
- C. See Adding Sequences.
- D. The n<sup>th</sup> triangular number can be calculated by the equation:

$$\sum_{i=1}^{n} i = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

- E. There are various ways of using triangular numbers:
  - 1. Calculating the n<sup>th</sup> triangular number.
    - a. Use the formula:  $\binom{n(n+1)}{2}$
  - 2. Finding the triangular number from a given number.
    - a. Solve the formula:  $\frac{n(n+1)}{2} = x$ , where x is the number.
  - 3. Adding successive triangular numbers.
    - a. Use the formula:  $n^2$ , where n is the larger of the two triangular numbers.

NOTE: This is because 2 triangles back to back make a square.

- 4. Subtracting 2 successive triangular numbers.
  - a. The answer is simply: n, where n is the larger of the two triangular numbers.
- F. Below are a few examples:
  - Ex [1] Find the 10<sup>th</sup> triangular number.
    - a. Using the formula we get:  $\frac{(10)(11)}{2}$ .
    - b. This reduces to (5)(11) or 55.
    - c. The answer is 55.
  - Ex [2] The 8<sup>th</sup> plus the 9<sup>th</sup> triangular number is \_\_\_\_\_.
    - a. Since 9 is the largest, we use:  $9^2$  or 81.
    - b. The answer is 81.

Ex [3] The 11<sup>th</sup> minus the 12<sup>th</sup> triangular number is \_\_\_\_\_.

- a. We just use the largest number, which is 12.
- b. However, the answer is going to be negative since the  $12^{th}$  triangular number is larger than the  $11^{th}$ .
- c. The answer is -12.