Solving $x^2 - y^2 = a^3$, Where x And y Are Triangular Numbers

A. In this type of problem we are squaring triangular numbers which means (see *Triangular Numbers*):

$$\left(\frac{n(n+1)}{2}\right)^2$$

B. However, this is the same formula for adding $1^3 + 2^3 + \ldots + n^3$ (see *Adding Cubes*).

C. Therefore, $x^2 - y^2 = a^3$ means:

$$(1^3 + 2^3 + \ldots + x^3) - (1^3 + 2^3 + \ldots + y^3) = a^3$$

The only way for this to be true is if x = a and y = a - 1

D. So, to solve this problem, choose x = $(a)^{th}$ triangular number and choose y = $(a - 1)^{th}$ triangular number.

E. Examples:

Ex [1] $x^2 - y^2 = 6^3$, x and y are negative triangular numbers, then x = ______.

a. For this problem we need x = 6^{th} triangular number and y = 5^{th} triangular number.

b. Since we are only concerned with x, we need the 6^{th} triangular number which is 21. Since the answer has to be negative, the answer is -21.

c. If the question had asked for y, the answer would have been the 5^{th} triangular number which is 15. Since the answer has to be negative, it would be -15.

Ex [2] $x^2 - y^2 = 512$ and x,y are triangular numbers, then y = ____.

a. For this problem you should know that $512 = 8^3$. So x is equal to the 8^{th} triangular number and y is equal to the 7^{th} triangular number.

b. The answer is the 7^{th} triangular number or 28.

c. If the question had asked for x, the answer would be the 8^{th} triangular number or 36.