

Logarithms:

- A. Logarithms, usually called logs, can be a little complicated since there are so many properties to learn. To become good at the log problems requires lots of practice.
- B. There are 2 basic logs: \log_n and \log_e (which is written as \ln and means "natural log").

1. In general, $\log_b x = n$ means $b^n = x$ and $\ln x = n$ means, $e^n = x$.
2. If no 'b' is given, then it is assumed the value 10.

Ex [1] $\log 1000 = \underline{\hspace{2cm}}$

- a. Since no value 'b' is given, $b = 10$.
- b. The problem is $10^x = 1000$. For this to be true, $x = 3$.
- c. The answer is 3.

Ex [2] $\log_4 x = 3/2$, then $x = \underline{\hspace{2cm}}$

- a. The answer is $4^{3/2} = x$ or $8 = x$. The answer is 8.

Ex [3] $\log_b 81 = \log_2 16$, then $b = \underline{\hspace{2cm}}$

- a. This means $b^{\log_2 16} = 81$. So the first thing to do is to find $\log_2 16$ which means $2^x = 16$. So $x = 4$. So $\log_2 16 = 4$.
- b. So substituting we get $b^4 = 81$. So $b = 3$.
- c. The answer is 3.

C. Properties Of Logs:

1. Below are the common properties of logs. All of these should be memorized and you should be very familiar with knowing how to manipulate them.

$\log_n n = 1$	$\log_n 1 = 0$	$\log a + \log b = \log ab$
$\log a - \log b = \log \frac{a}{b}$	$n^{\log_n a} = a$	$\log_n \left(\frac{1}{x}\right) = -\log_n x$
$\log_n a \times \log_a n = 1$	$\log n^a = a \times \log n$	$\log_n x = \frac{\log x}{\log n}$

2. There might be other properties, but these are the ones that show up on the number sense tests the most.

D. Examples Taken From Past UIL Tests

Ex [4] $\log_4 27 \div \log_4 3 = \underline{\hspace{2cm}}$

- In this problem, if we can make the 1st term be $\log_4 3$ then the problem would be easy.
- Change $\log_4 27$ to $\log_4 3^3 = 3 \cdot \log_4 3$.
- So now we have $3 \times \log_4 3 \div \log_4 3 = 3 \times 1 = 3$.
- The answer is 3.

Ex [5] $\ln 5 + \ln 8 = \ln 10 - \ln x$, then $x = \underline{\hspace{2cm}}$

- $\ln 5 + \ln 8 = \ln 5 \cdot 8 = \ln 40$.
- $\ln 10 - \ln x = \ln \frac{10}{x}$.
- So we have $\ln 40 = \ln \frac{10}{x}$. The only way for this to be true is if $40 = \frac{10}{x}$. So, solving for x we get $x = \frac{1}{4}$.
- The answer is $\frac{1}{4}$.

Ex [6] $(\log_2 7)(\log_7 4) = \underline{\hspace{2cm}}$

- We know that $(\log_b a)(\log_a b) = 1$. So we need to change the 2nd term to be $\log_7 2$ to use this property.
- Change $\log_7 4$ to $\log_7 2^2 = 2 \cdot \log_7 2$.
- So we have $2 \times \log_2 7 \times \log_7 2 = 2 \times 1 = 2$.
- The answer is 2.