Limits:

A. Limits can be calculated very quickly and are closely related to *horizontal asymptotes*.

B. There are many ways limits can be used:

1. $\lim_{x \to n} f(x) = f(n)$

Ex [1] $\lim_{x \to 2} x^2 - 4 =$ _______

a. The answer is f(2) or $2^2 - 4 = 0$.

2. $\lim_{x \to n} \frac{f(x)}{g(x)} = \frac{f(n)}{g(n)}$

a. If g(n) = 0 and f(n) does not, then the limit is undefined.

b. If g(n) = 0 and f(n) = 0, then use *L'Hopital's Rule*. See Below.

c. If g(n) does not equal 0, then the answer is $^{f(n)}/_{g(n)}$

Ex [2] $\lim_{x \to 4} \frac{3x^2 + 1}{x - 2} =$ _________

a. Since g(4) = 2 and not 0, the answer is $^{f(4)}/_{g(4)} = {}^{3(16)+1}/_{2} = {}^{49}/_{2}$.

3. $\lim_{x \to \infty} \frac{f(x)}{g(x)}$

a. Problems like these have the same basic rules as *horizontal asymptotes*.

b. If the degree of the numerator is greater than the degree of the denominator the answer is ∞.

c. If the degree of the numerator is less than the degree of the denominator the answer is 0.

d. If the degrees are the same the answer is $^{a}/_{d}$ where a is the coefficient in front of the highest degree in the numerator and d is the coefficient in front of the highest degree in the denominator.

Ex [3] $\lim_{x \to \infty} \frac{3x^3 - 2}{4x^4 - 1} =$ __________

a. Since the degree of the denominator is 4 and the degree of the numerator is 3 and 4>3, the answer is 0.

Ex [4] $\lim_{x \to \infty} \frac{4x^2 - 3}{2x^2 - 1} =$ __________

a. Since the degrees are the same the answer is $^4/_2 = 2$.

C. L' Hopital's Rule

1. L'Hopital's Rule is a very handy rule that makes solving problems very easy. It states that if when taking the limit of $^{f(x)}/_{g(x)}$ you get the value of $^0/_0$ or ∞/∞, then the limit is equal to $^{f'(x)}/_{g'(x)}$. (See *derivatives*)

Ex [5] $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 - 4} =$ ________

a. If I plug in 2 into the equation I get $^0/_0$, which means I can use this rule.

b. The derivative of the numerator is 2x - 4.

c. The derivative of the denominator is 2x.

d. Therefore the limit is equal to $^{2(2)-4}/_{2(2)} = {}^0/_4 = 0$.

e. The answer is 0.

Ex [6] $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} =$ ________

a. Notice, that when I plug in 3, I do in fact get $^0/_0$.

b. The derivative of the numerator is 2x - 4.

c. The derivative of the denominator is 2x - 2.

d. Therefore, the limit is equal to $^{2(3)-4}/_{2(3)-2} = {}^2/_4 = {}^1/_2$.

e. The answer is $^1/_2$.

Ex [7] $\lim_{x \to 0} \frac{\sin 3x}{x} =$ _______

a. In this problem, we get $^0/_0$, so we can use this rule.

b. The derivative of sin 3x = 3 cos 3x.

c. The derivative of x = 1.

d. So $^{3*\cos 3(0)}/_1 = 3 \times 1 = 3$.

e. The answer is 3.

D. Instead of using L' Hopital's Rule, you can factor the equations and cancel something out, but this rule is much faster and easier.