Derivatives:

A. A derivative is calculated the exact opposite to that of an *integral*.

B. A function's derivative is basically the equation for the slope of the original function. Derivatives are usually expressed by: f '(x) or y' or $^{dy}/_{dx}$. f '(x) or y' is the first derivative. f "(x) or y" is the second derivative and so on.

C. Below are the basic rules for computing derivatives.

1. The derivative of a constant is 0.

   Ex [1] $^{dy}/_{dx}\ 5 = 0$.

2. The derivative of x^n is $n*x^{n-1}$.

   Ex [2] $^{dy}/_{dx}\ 3x^2 = 6x$.

3. The derivative of $f(x) \pm g(x) = f'(x) \pm g'(x)$

   Ex [3] $^{dy}/_{dx}\ 6x^3 + 4x^2 + 8x - 4 =$ _________

   a. This rule means you can take each term separately.

   b. So this becomes $^{dy}/_{dx}\ 6x^3 + {}^{dy}/_{dx}\ 4x^2 + {}^{dy}/_{dx}\ 8x - {}^{dy}/_{dx}\ 4 = 18x^2 + 8x + 8 - 0$.

   c. The answer is $18x^2 + 8x + 8$.

4. The derivative of $[f(x)]^n$ is $f'(x)*[f(x)]^{n-1}$.

   Ex [4] $^{dy}/_{dx}\ (3x^2+5x+2)^3 =$ ________

   a. You always want to work from the inside out.

   b. The first step is to take the derivative of the inside first. So $^{dy}/_{dx}\ 3x^2 + 5x + 2 = 6x + 5$. This represents f '(x).

   c. Now, we need the derivative of the outside which is $3(3x^2+5x+2)^2$.

   d. Now, multiplying these two values together gives:

   $$(6x+5)*3*(3x^2+5x+2)^2 \text{ or } (18x+15)(3x^2+5x+2)^2.$$

5. The derivative of $f(x)*g(x) = f'(x)g(x)+f(x)g'(x)$.

    a. This type of problem will probably not be found on a number sense test.

    Ex [5] $\frac{dy}{dx}(3x-4)(x^2-3) =$ _________

    a. First, multiply the derivative of the first times the second. So we get: $\frac{dy}{dx}3x - 4 = 3$. So $3(x^2-3)$ is the first term.

    b. Next, multiply the first term times the derivative of the second. So we get: $\frac{dy}{dx}x^2 - 3 = 2x$. So we get $2x(3x-4)$ for the second term.

    c. The answer is $3(x^2-3)+2x(3x-4)$.

D. Common Derivatives:

1. The derivative of $\sin(f(x)) = f'(x)*\cos(f(x))$.
2. The derivative of $\cos(f(x)) = -f'(x)*\sin(f(x))$.
3. The derivative of $\tan(f(x)) = f'(x)*\sec^2(f(x))$.
4. The derivative of $e^{f(x)} = f'(x)*e^{f(x)}$.
5. The derivative of $\ln(f(x)) = f'(x)*\frac{1}{f(x)}$.

E. Examples

Ex [1] If $f(x) = \sin 4x$, then $f'(\pi/4) =$ _______

a. First, take the derivative of 4x which is 4.

b. The derivative of sin x is cos x, so the deriv ative of $\sin 4x = 4\cos 4x$.

c. Plugging in $x = \pi/4$, we get $4\cos\pi = 4(-1) = -4$.

d. The answer is -4.

Ex [2] Find the slope of the tangent line of $y = (x-2)^3$ at the point $x = 4$.

a. Since a derivative is the slope of the tangent line, we need to find y' and use the value $x = 4$ to find the slope.

b. $y' = 3(x-2)^2$, so plugging in $x=4$ we get $3(2^2) = 12$.

c. The answer is 12.

Ex [3] If $f(x) = 4x^3 - 12x^2 + 4x - 3$, then $f''(2) =$ ______

a. For this problem we are looking for the second derivative. So we need to find the derivative of the derivative, or just take the derivative twice.

b. $f'(x) = 12x^2 - 24x + 4$

c. So $f''(x) = 24x - 24$. $f''(2) = 24(2) - 24 = 24$.

d. The answer is 24.