Vectors:

A. There are many different ways of working with vectors and vector spaces. The following ways are only possibilities for a number sense test. It is possible they will never show up.

B. Dot Product

1. The dot product of a vector, $\mathbf{x} \cdot \mathbf{y}$, is defined as being:

$$\sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$

Ex [1] The dot product of the vectors (3,4) and (-1,5) is _____

a. The answer is 3(-1) + 4(5) = 17.

Ex [2] The dot product of the vectors (0,1,1) and (5,4,-3) is _____

a. The answer is 0(5) + 1(4) + 1(-3) = 1.

C. Cross Product

1. To find the cross product of 2 vectors, you need to be familiar wit h *determinants* of matrices.

2. The cross product of 2 vectors (of dimension 2), **u** x **v**, is defined as:

$$\mathbf{u} \times \mathbf{v} = \det\begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} = u_1 v_2 - u_2 v_1$$

Ex [1] The cross product of the vectors (3,5) and (-1,3) is _____

a. The answer is 3(3) - 5(-1) = 14.

Ex [2] The cross product of the vectors (-1,a) and (-2,4) is 6, so a = _____

a. This problem is a little more complicated. Just set up an algebraic expression.

b. -1(4) - (-2)a = 6 or -4 + 2a = 6. Solving for a we get a = 5.

c. The answer is 5.

D. Norm

1. The norm of a vector, $\|\mathbf{u}\|$, is defined as being:

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$$

2. Knowing *Pythagorean Triples* will be helpful.

Ex [1] Find the norm of the vector (0,12,5) _____

a. You should know the Pythagorean triple (5,12,13). The answer is 13.

b. If you don't know this, you can see that $\sqrt{0^2 + 12^2 + 5^2} = \sqrt{169} = 13$.