Inverse Functions:

A. An inverse of a function, usually written as $f^{-1}(x)$, is a reflection of the original function, f(x), around the line y = x. Basically, every x value is changed to a y value and every y value is change to an x value.

B. To find a function's inverse, simply switch the x and y variables and solve for y.

C. Number sense usually deals with inverses in one of three ways:

1. The inverse of the function: f(x) = ax + b, is: $f^{-1}(x) = {}^{x-b}/_{a}$ or $^{1}/_{a}x - {}^{b}/_{a}$.

Ex [1] If f(x) = 2x + 4 and $f^{-1}(x)$ = ax + b, then a = ____.

a. For 'a' the answer is $^{1}/_{a}$ or $^{1}/_{2}$.

b. If the question had asked for 'b' then the answer would be $-^{b}/_{a}$ or $-^{4}/_{2} = -2$.

Ex [2] If f(x) = x - 5, then $f^{-1}(3)$ = ______.

a. In this problem we have to first find $f^{-1}(x)$ then find $f^{-1}(3)$.

b. $f^{-1}(x) = x + 5$. So $f^{-1}(3) = 3 + 5 = 8$.

c. The answer is 8.

2. The inverse of the function: $f(x) = \frac{ax+b}{cx+d}$ is $\frac{-dx+b}{cx-a}$. Notice, b and c remain the same and d and a are switched and their signs are changed.

Ex [1] If $f(x) = \frac{3x+4}{2x-4}$ and $f^{-1}(x) = \frac{ax+b}{cx+d}$ then c = _______

a. In this problem 'c' stays the same, so c = 2.

b. If the problem had asked for 'a' the answer would have been -(-4) or 4. 'b' stays the same, so 'b' = 4. 'd' would be -3.

Ex [2] If $f(x) = \frac{3x+5}{x-3}$ then $f^{-1}(4) =$ _______

a. In this problem we have to find $f^{-1}(x)$ first.

b. If you notice, $f^{-1}(x) = f(x)$. There is no change. So all we have to do is plug in 4 to the equation. Doing so gives $^{3(4)+5}/_{4-3} = 17$.

c. The answer is 17.

3. For this last type of problem, you need to switch the x and y variables and solve for y to find the inverse.

Ex [1] If $3x - 4y = 2$ and $y^{-1} = ax + b$, then a = ____.

a. In this case switch the x and y variables and solve for y. So we get $3y - 4x = 2$.

b. Solving for y we get $3y = 4x + 2$ or $y = {}^{4}/_{3}x + {}^{2}/_{3}$.

c. The answer is $^{4}/_{3}$.

Ex [2] If $5y + x - 2 = 0$, and $y^{-1} = ax + b$, then b = _____.

a. Switching the x and y variables we get $5x + y - 2 = 0$.

b. Solving for y we get: $y = -5x + 2$.

c. The answer is 2.