Horizontal And Vertical Asymptotes And Discontinuities:

A. An asymptote is a line that the graph of a function approaches but never reaches. There are two main types of asymptotes: Horizontal and Vertical. (Note: There are others but number sense only focuses on these two.)

B. A discontinuity is a break in the graph. Some discontinuities can be "removed" and are called "removable discontinuities".

C. Vertical Asymptotes

1. To find a vertical asymptote, set the denominator equa l to 0 and solve for x. If this value, a, is not a *removable discontinuity*, then x=a is a vertical asymptote.

Ex [1] The graph of $\frac{x^2-5x+4}{x^2-1}$ has a vertical asymptote at x = ________.

a. To find the vertical asymptotes we need to set the denominator = 0 and solve.

b. Doing so gives: $x^2 - 1 = 0$ which is (x-1)(x+1)=0. This gives the values of x=1 and x=-1.

c. However, x=1, is a removable discontinuity, so the answer is x = -1.

D. Horizontal Asymptotes

1. To find a function's horizontal asymptotes, there are 3 situations.

a. The degree of the numerator is higher than the degree of the denominator.

1. If this is the case, then there are no horizontal asymptotes.

b. The degree of the numerator is less than the degree of the denominator.

1. If this is the case, then the horizont al asymptote is y=0.

Ex [1] The equation $\frac{x^2-5x+4}{x^3+5}$ has a horizontal asymptote of y = _____

a. Notice, the degree of the numerator is 2 and the degree of the denominator is 3 so the answer is 0.

c. The degree of the numerator is the same as the degree of the denominator.

1. If this is the case, then the horizontal asymptote is y = $^{a}/_{d}$ where a is the coefficient in front of the highest degree in the num erator and d is the coefficient in front of the highest degree in the denominator.

Ex [2] The equation $\frac{3x^4 - 5x^2 + 4}{2x^4 - 2x^3 + 5}$ has a horizontal asymptote of y = ______

a. Notice, the degree of the numerator and the denominator are both 4.

b. The answer is $^{3}/_{2}$.

E. Removable Discontinuities

1. A discontinuity is a part of the graph that is undefined at a particular point, but there is no asymptote at that point. A removable discontinuity is when you can factor out a term in the numerator and factor out the same term in the denominator, thus canceling each other out.

a. Let's go back to Ex [1] in section C.

Ex [1] The graph of $\frac{x^2 - 5x + 4}{x^2 - 1}$ has a removable discontinuity at x = ________.

a. In this example, we can factor the numerator to (x -1)(x-4).

b. We can factor the denominator to (x -1)(x+1).

c. Since both the numerator and denominator have the term (x -1), this becomes a removable discontinuity. So setting this equal to 0, we get x=1 is a removable discontinuity.